

STOCHASTIC MODELLING OF SOLVENT PRODUCTION BY C. ACETOBUTYLICUM P262 IN FERMENTATION PROCESS

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Abstract

A typical batch fermentation process can be better presented and understood via stochastic delay differential equations, SDDEs. SDDEs represent a mathematical formulation of the system that behave in the presence of time delay and noise. In fact, there are two important features that control the mechanism of the fermentation process namely time delay and the system is continually subject to the effects of random, which is referred as noise. This research was conducted to model jointly time delay and random effects of solvent production by *C. acetobutylicum* P262 in batch fermentation process using SDDEs. Euler method is applied to simulate numerical solution of the model. Experimental data collected from the batch fermentation process is used for model verification.

Introduction

There are two important features that control the mechanism of the fermentation process namely time delay and noise. The presence of time delay is a consequence of the fact that initially microbial are in the process of adapting themselves to the new environment, thus no growth occur. The microbe is said to be in a lag phase. At the end of the lag phase, the population of microorganisms is well-adjusted to the new environment, cells multiply rapidly and cell mass doubles regularly with time. Hence, exponential phase arise. As time evolves, the system is subjected to an intrinsic variability of the competing within species which is referred as noise. Nutrient level and toxin concentration achieves a value which unable to sustain the maximum growth rate and the microbe is said to be in stationary phase. By considering the entire processes involves, it is practical to model the batch fermentation process via SDDEs.

Fermentation Process

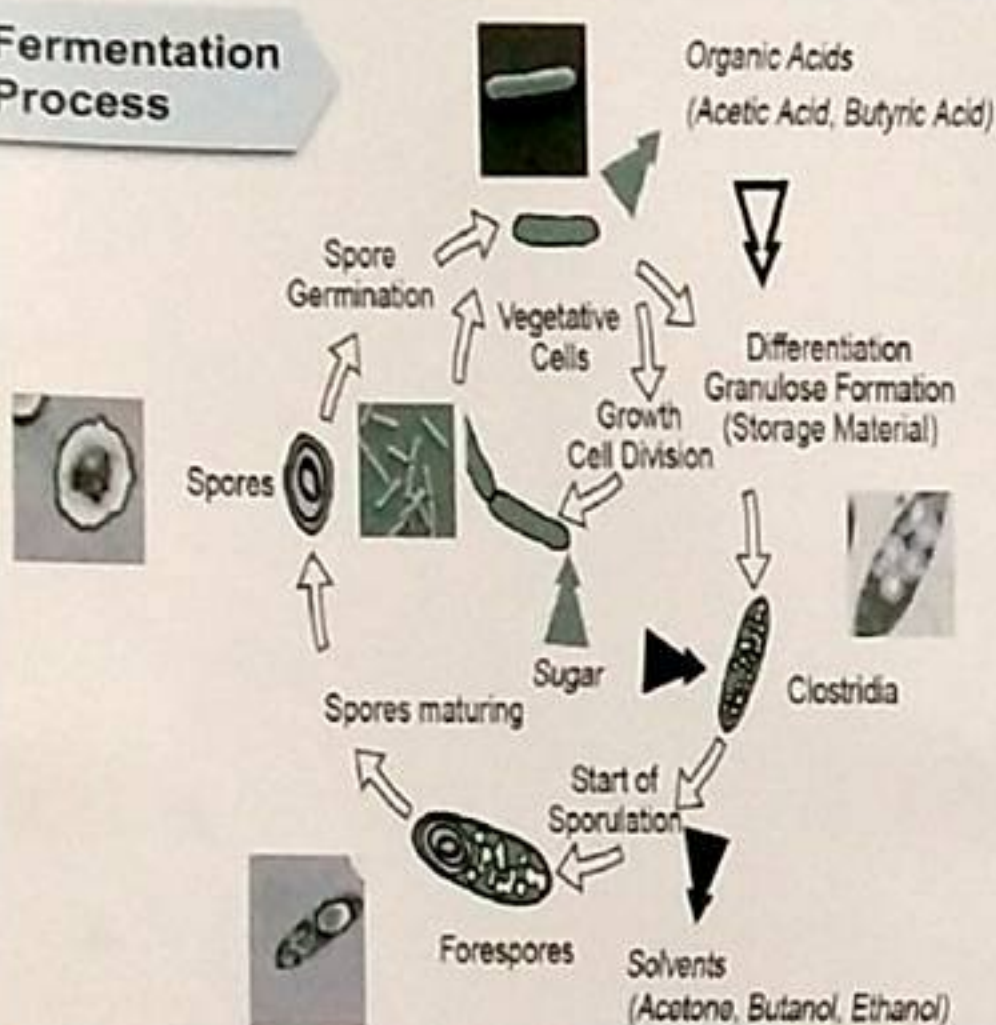


Figure 1 : Cell cycle of *C. acetobutylicum*
(source: Schuster et al. 1998)

Methodology

Deterministic Model of Cell Growth

$$dx(t) = \mu_{\max} \left(1 - \frac{x(t)}{x_{\max}} \right) x(t) dt, t \in [t_0, T]$$

Stochastic Model of Cell Growth

$$dx(t) = \mu_{\max} \left(1 - \frac{x(t-r)}{x_{\max}} \right) x(t) dt + \sigma x(t) dW(t), t \in [-r, T]$$

$$x(t) = \Phi(t), t \in [-r, t_0]$$

Luedeking - Piret Equations

$$dA(t) = ax(t)dt$$

$$dB(t) = bx(t)dt$$

Stochastic Model

| | |
|--------------|---|
| $x(t)$ | Cell concentration (g/L) |
| x_{\max} | Maximum cell growth |
| μ_{\max} | Maximum specific growth rate |
| σ | Diffusion coefficient |
| $W(t)$ | Wiener process |
| t | Time (h) |
| r | Time delay (h) |
| $A(t)$ | Acetone concentration (g/L) |
| $B(t)$ | Butanol concentration (g/L) |
| e | Growth associated coefficient for acetone concentration (g substrate/ g cell) |
| b | Growth associated coefficient for butanol concentration (g substrate/ g cell) |

Nomenclature

Results

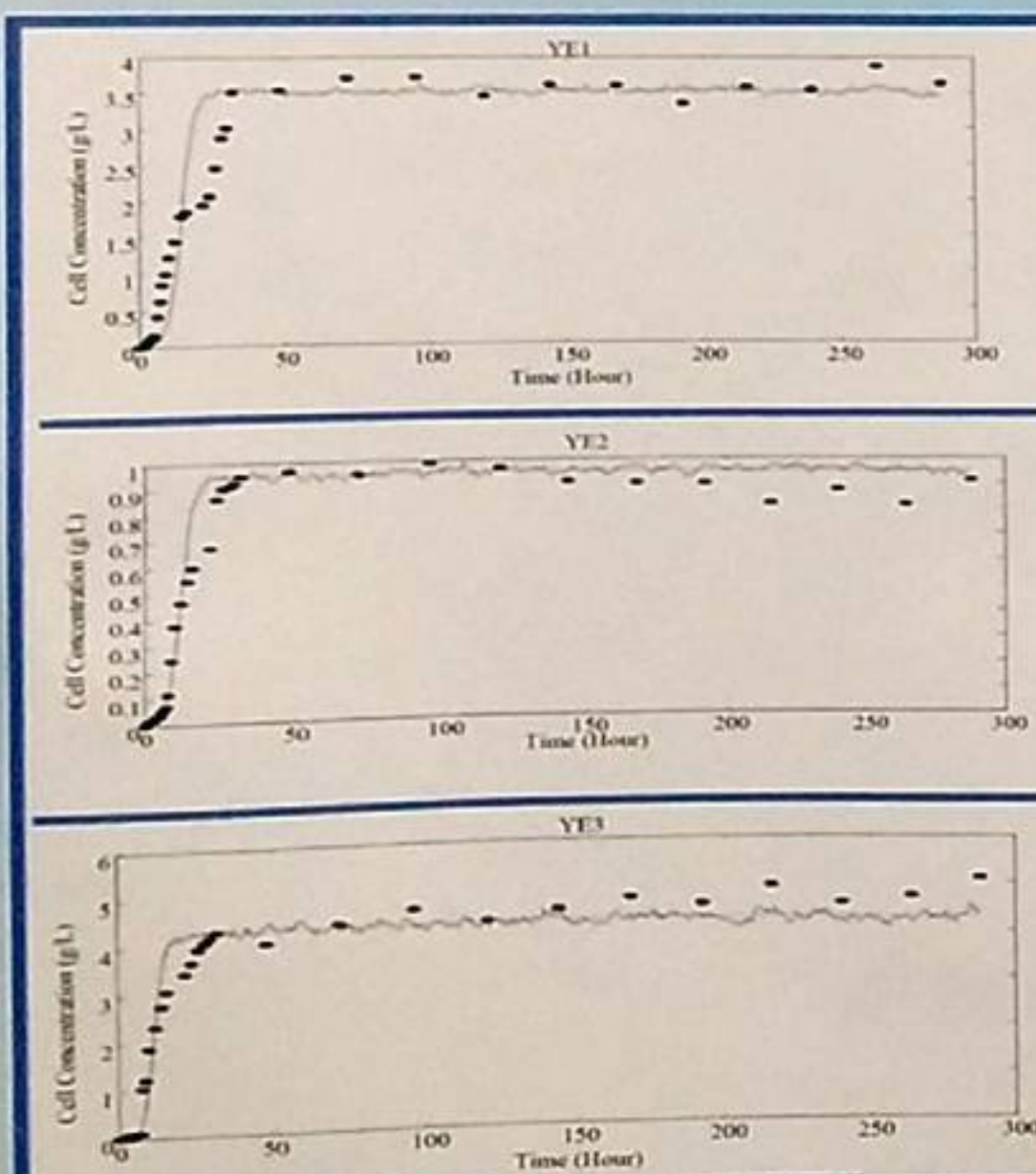


Figure 2: Cell Concentration

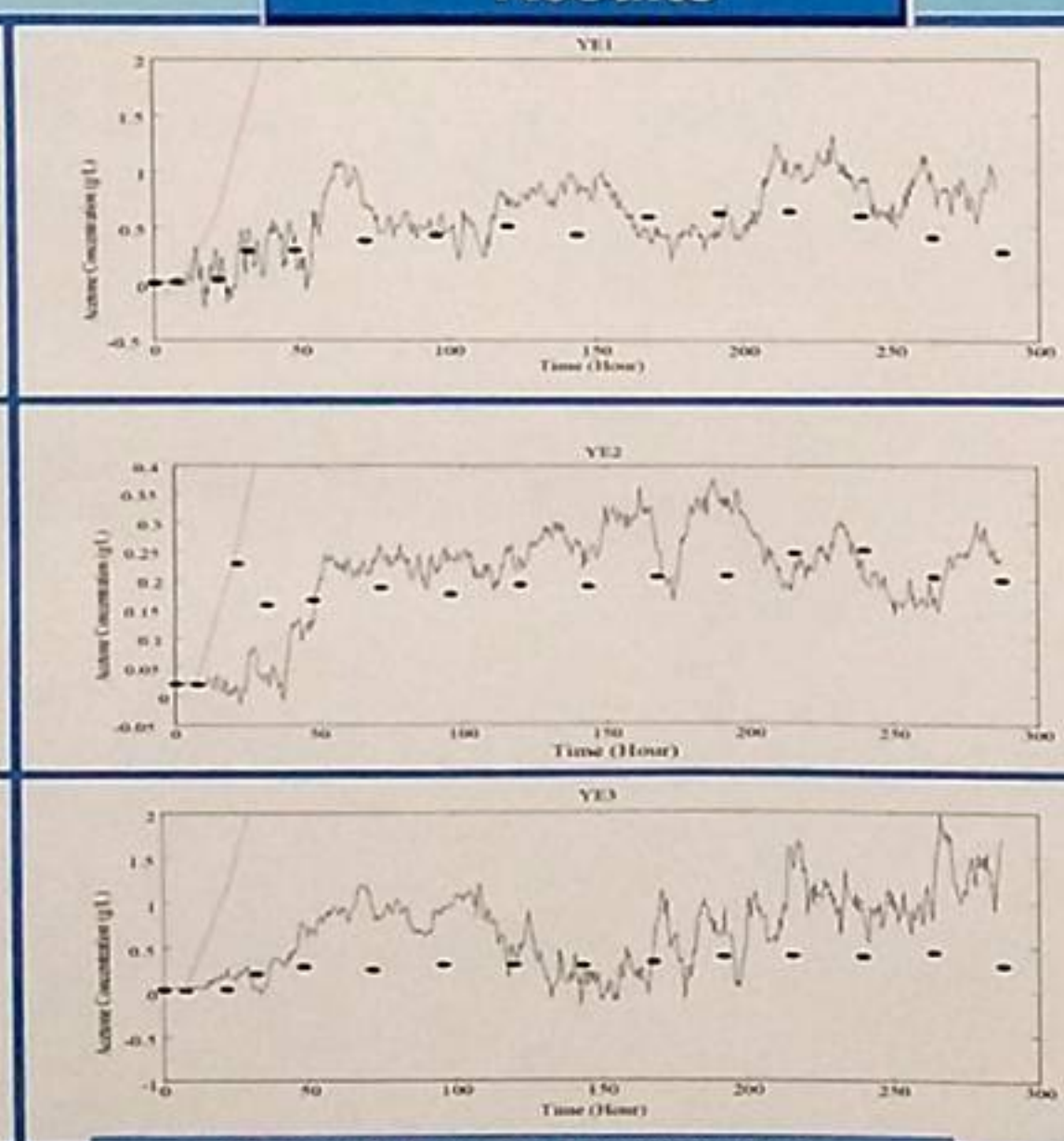


Figure 3: Acetone Concentration

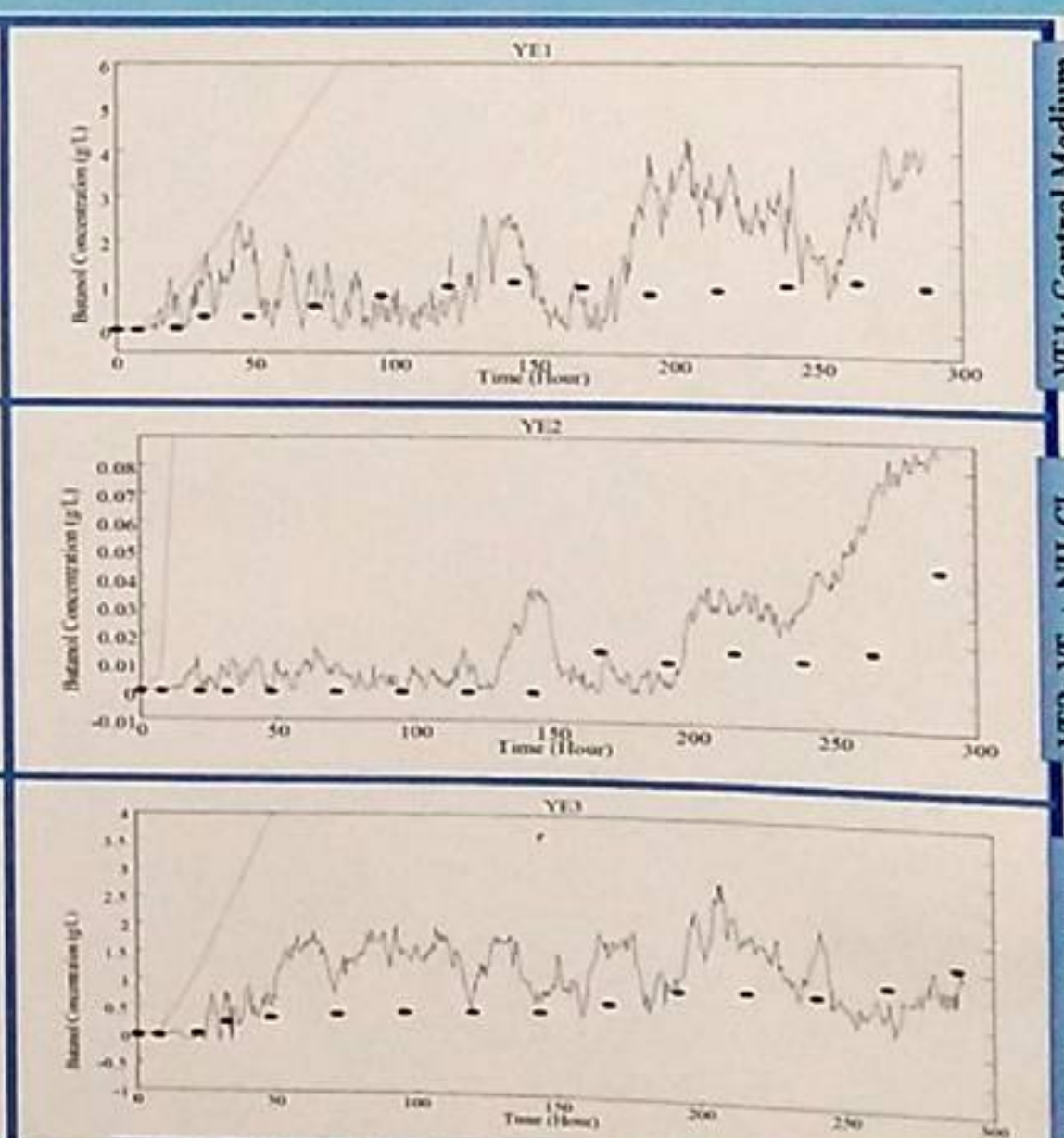


Figure 4: Butanol Concentration

Legend
— Deterministic Model
— Stochastic Model
• Experimental Data

Table 1: Parameter Estimation Values

| Kinetic Parameter | YE1 | YE2 | YE3 |
|--------------------|--------|--------|--------|
| $\hat{\mu}_{\max}$ | 0.4848 | 0.5056 | 0.6354 |
| $\hat{\sigma}$ | 0.0028 | 0.0127 | 0.0052 |
| \hat{x}_{\max} | 3.5250 | 0.9490 | 4.2950 |
| \hat{b} | 0.1076 | 0.0414 | 0.0835 |
| \hat{e} | 0.2596 | 0.0072 | 0.2431 |

Table 2: RMSE of Stochastic & Deterministic Models for YE1, YE2 and YE3

| Mathematical Model | YE1 | YE2 | YE3 |
|--|--------|--------|--------|
| Stochastic model with time delay and Luedeking-Piret equations | 0.4356 | 0.0905 | 0.4586 |
| | 0.2042 | 0.1859 | 0.0666 |
| | 0.2312 | 0.0447 | 0.1540 |
| Deterministic model and Luedeking-Piret equations | 0.5483 | 0.1058 | 0.5420 |
| | 1.1262 | 0.6051 | 3.4364 |
| | 3.4007 | 2.0022 | 9.4259 |

Conclusion

The numerical solution of stochastic delay differential equations for cell growth and Luedeking-Piret equations of solvent production describe the experimental data with more adequacy as indicated by low values of RMSE for YE1, YE2 and YE3. Thus, we can conclude that the cell growth and solvent production in batch fermentation process can be better presented and understood via stochastic delay differential equations.

References

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Acknowledgement : We would like to thank the Ministry of Higher Education (MOHE) and RMC UMP for the RACE Vote No. RDU121303.